Period 1, Sept 23, 2024

$\frac{x^7}{x^4} = x^{7-4} = x^3$

23. $y = \frac{4x^{3/2}}{x^1}$

$y = 4x^{3/2-1} = 4x^{\frac{1}{2}}$

$\frac{dy}{dx} = 4 \cdot \frac{1}{2} x^{\frac{1}{2}-1} = 2x^{-\frac{1}{2}} = \frac{2}{\sqrt{x} \cdot \sqrt{x}} = \frac{2\sqrt{x}}{x}$

29. $f(x) = \frac{3x-1}{\sqrt{x}} = \frac{3x}{\sqrt{x}} - \frac{1}{\sqrt{x}} = 3x^{1-\frac{1}{2}} - 1x^{-\frac{1}{2}} = 3x^{\frac{1}{2}} - x^{-\frac{1}{2}}$

$F'(x) = 3 \cdot \frac{1}{2} x^{\frac{1}{2}-1} - 1 \cdot -\frac{1}{2} x^{-\frac{1}{2}-1} = \frac{3}{2} x^{-\frac{1}{2}} + \frac{1}{2} x^{-\frac{3}{2}}$

$\sqrt{x^3} = x\sqrt{x}$

$\frac{3 \cdot x}{2\sqrt{x} \cdot x} + \frac{1}{2\sqrt{x^3}} = \frac{3\sqrt{x^2}+1}{2\sqrt{x^3}} = \frac{3x+1}{2x\sqrt{x}}$

$F(x) = \frac{3x-1}{\sqrt{x}}$

$F'(x) = \frac{3(\sqrt{x}) - (3x-1) \cdot \frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$

$\frac{3\sqrt{x} \cdot 2\sqrt{x}}{1 \cdot 2\sqrt{x}} - \frac{(3x-1)}{2\sqrt{x}} = \frac{\frac{6x}{2\sqrt{x}} - \frac{3x-1}{2\sqrt{x}}}{x}$

$\frac{3x+1}{2\sqrt{x}} \cdot \frac{1}{x}$

$\frac{3x+1}{2x\sqrt{x}}$

$\frac{6x-3x+1}{2\sqrt{x}} \div \frac{x}{1}$

31. $h(s) = (s^3-2)^2 = (s^3-2)(s^3-2) \Rightarrow h(x) = (x^3-2)(x^3-2) = x^6 - 2x^3 - 2x^3 + 4$

$x^6 - 4x^3 + 4$

$h(x) = (x^3-2)(x^3-2)$

$h'(x) = 3x^2(x^3-2) + (x^3-2)(3x^2)$

$2 \cdot 3x^2(x^3-2)$

$6x^2(x^3-2)$

$6x^5 - 12x^2$

$h(x) = x^6 - 4x^3 + 4$

$h'(x) = 6x^5 - 12x^2 + 0$

$6x^2(x^3-2)$

52. $f(x) = \sin x \cos x$

$F'(x) = \cos x \cdot \cos x + \sin x(-\sin x)$

$\cos^2 x - \sin^2 x$

$\cos 2x$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\cos 2x$

$\cos(x+x) = \cos x \cos x - \sin x \sin x$

$\cos 2x = \cos^2 x - \sin^2 x$

30. $f(x) = \sqrt[3]{x}(\sqrt{x}+3) = x^{\frac{1}{3}}(x^{\frac{1}{2}}+3) = x^{\frac{2}{6}+\frac{3}{6}} + 3x^{\frac{1}{3}} = x^{\frac{5}{6}} + 3x^{\frac{1}{3}}$

$F'(x) = \frac{1}{3}x^{\frac{1}{3}-1}(x^{\frac{1}{2}}+3) + x^{\frac{1}{3}}\left(\frac{1}{2}x^{\frac{1}{2}-1}+0\right)$

$\frac{1}{3}x^{-\frac{2}{3}}(x^{\frac{1}{2}}+3) + x^{\frac{1}{3}}\cdot\frac{1}{2}x^{-\frac{1}{2}}$

$\frac{1}{3}\cdot\frac{1}{x^{2/3}}\cdot\frac{(x^{\frac{1}{2}}+3)}{1} + x^{\frac{1}{3}}\cdot\frac{1}{2x^{1/2}}$

$\frac{x^{\frac{1}{2}}+3}{3x^{2/3}} + \frac{x^{\frac{1}{3}}}{2x^{\frac{1}{2}}}$

$\frac{(\sqrt{x}+3)\sqrt[3]{x}}{3\sqrt[3]{x^2}\cdot\sqrt[3]{x}} + \frac{\sqrt[3]{x}\sqrt{x}}{2\sqrt{x}\cdot\sqrt{x}}$

$\frac{x^{\frac{1}{2}}\cdot x^{\frac{1}{3}} + 3\cdot x^{\frac{1}{3}}}{3x} + \frac{x^{\frac{1}{3}}\cdot x^{\frac{1}{2}}}{2x} = \frac{(x^{\frac{5}{6}}+3x^{\frac{1}{3}})\cdot 2}{3x\cdot 2} + \frac{x^{\frac{5}{6}}\cdot 3}{2x\cdot 3} = \frac{2x^{\frac{5}{6}}+6x^{\frac{1}{3}}+3x^{\frac{5}{6}}}{6x}$

$2+\frac{1}{2} = \frac{5}{2}$

$\frac{5x^{\frac{5}{6}}+6x^{\frac{1}{3}}}{6x} =$

$F(x) = x^{\frac{5}{6}} + 3x^{\frac{1}{3}}$

$F'(x) = \frac{5}{6}x^{\frac{5}{6}-1} + 3\cdot\frac{1}{3}x^{\frac{1}{3}-1}$

$\frac{5}{6}x^{-\frac{1}{6}} + x^{-\frac{2}{3}}$

$\frac{5\sqrt[6]{x^5}}{6\sqrt[6]{x}\sqrt[6]{x^5}} + \frac{1}{\sqrt[3]{x^2}\cdot\sqrt[3]{x}}$

$\frac{5\sqrt[6]{x^5}}{6x} + \frac{6\cdot\sqrt[3]{x}}{6\cdot x} = \frac{5\sqrt[6]{x^5}+6\sqrt[3]{x}}{6x}$

3. $h(t) = \sqrt{t}(1-t^2) = \sqrt{t} - t^2\sqrt{t} = t^{\frac{1}{2}} - t^{\frac{5}{2}}$

$h'(t) = \frac{1}{2}t^{\frac{1}{2}-1} - \frac{5}{2}t^{\frac{5}{2}-1} = \frac{1}{2\sqrt{t}} - \frac{5t^{3/2}}{2} = \frac{1}{2\sqrt{t}} - \frac{5t\sqrt{t}\sqrt{t}}{2\cdot\sqrt{t}}$

$\sqrt{t}\cdot\sqrt{t} = t$

$\frac{1-5t\cdot t}{2\sqrt{t}} = \frac{1-5t^2}{2\sqrt{t}}$

4. $g(s) = \sqrt{s}(s^2 + 8)$ $\Rightarrow g(x) = \sqrt{x}(x^2+8) = x^{\frac{1}{2}}\cdot x^2 + x^{\frac{1}{2}}\cdot 8 = x^{5/2} + 8x^{\frac{1}{2}}$

$g'(x) = \frac{5}{2}x^{\frac{5}{2}-1} + 8\cdot\frac{1}{2}x^{\frac{1}{2}-1} = \frac{5}{2}x^{3/2} + 4x^{-\frac{1}{2}} = \frac{5x\sqrt{x}\sqrt{x}}{2\cdot\sqrt{x}} + \frac{4}{2\sqrt{x}}$

$\frac{5x\cdot x}{2\sqrt{x}} + \frac{8}{2\sqrt{x}} = \frac{5x^2+8}{2\sqrt{x}}$

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35. $f(x) = (2x^3 + 5x)(x - 3)(x + 2)$

36. $f(x) = (x^3 - x)(x^2 + 2)(x^2 + x - 1)$

$x^2 + c^2$

35. $F'(x) = (6x^2+5)(x-3)(x+2) + (2x^3+5x)(1)(x+2) + (2x^3+5x)(x-3)(1)$

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36. $F(x) = (x^3-x)(x^2+2)(x^2+x-1)$

$F'(x) = (3x^2-1)(x^2+2)(x^2+x-1) + (x^3-x)(2x)(x^2+x-1) + (x^3-x)(x^2+2)(2x+1)$

$2\sin x\cos x = \sin 2x$

$2x\sin x\cos x = x\cdot 2\cdot\sin x\cdot\cos x$

$x\cdot\sin 2x$

51. $f(x) = x^2\tan x$

$F'(x) = 2x\cdot\tan x + x^2\cdot\sec^2 x$

$\frac{2x\sin x\cos x}{\cos x\cos x} + \frac{x^2}{\cos^2 x} = \frac{2x\sin x\cos x}{\cos^2 x} + \frac{x^2}{\cos^2 x} = \frac{x\cdot\sin 2x}{\cos^2 x} + \frac{x^2}{\cos^2 x}$

$\frac{x(\sin 2x + x)}{\cos^2 x}$

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54. $h(\theta) = 5\theta\sec\theta + \theta\tan\theta$

$h'(\theta) = 5\cdot\sec\theta + 5\theta\cdot\sec\theta\tan\theta + 1\cdot\tan\theta + \theta\cdot\sec^2\theta$

$= 5\sec\theta + \theta\sec^2\theta + 5\theta\sec\theta\tan\theta + \tan\theta =$

$\sec\theta(5 + \theta\sec\theta) + \tan\theta(5\theta\sec\theta + 1)$

$$h(x) = \frac{F(x)}{g(x)}$$

$$h'(x) = \frac{F'(x)\cdot g(x) - F(x)\cdot g'(x)}{(g(x))^2}$$

$$F(x) = \tan x = \frac{\sin x}{\cos x}$$

$$F'(x) = \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{(\cos x)^2}$$

$$F'(x) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$